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VECTORIZED SPARSE ELIMINATION(U) MICHIGAN UNIV ANN
ARBOR DEPT OF ELECTRICAL AND COMPUTER ENGINEERING
D A CALAHAN 01 MAR 84 AFOSR-TR-84-0367 AFOSR-80-0158

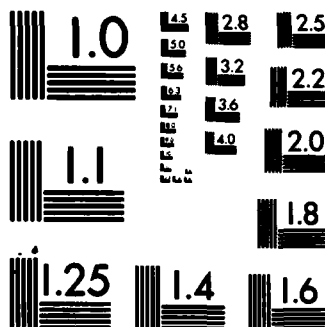
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Final Report
VECTORIZED SPARSE ELIMINATION

For Period 5/1/80 - 4/30/84

Grant AF-AFOSR 80-0158

SELECTED
MAY 17 1984
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March 1, 1984

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS										
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.										
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 84-0367										
4. PERFORMING ORGANIZATION REPORT NUMBER(S)			7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research										
6a. NAME OF PERFORMING ORGANIZATION University of Michigan		6b. OFFICE SYMBOL (If applicable)	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332										
6c. ADDRESS (City, State and ZIP Code) Department of Electrical & Computer Engineering, Ann Arbor MI 48109		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-80-0158											
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR		8b. OFFICE SYMBOL (If applicable) NM	10. SOURCE OF FUNDING NOS.										
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332		<table border="1"> <tr> <th>PROGRAM ELEMENT NO.</th> <th>PROJECT NO.</th> <th>TASK NO.</th> <th>WORK UNIT NO.</th> </tr> <tr> <td>61102F</td> <td>2304</td> <td>A3</td> <td></td> </tr> </table>				PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.	61102F	2304	A3	
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61102F	2304	A3											
11. TITLE (Include Security Classification) VECTORIZED SPARSE ELIMINATION													
12. PERSONAL AUTHOR(S) D.A. Calahan													
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM <u>1/5/80</u> TO <u>30/4/84</u>		14. DATE OF REPORT (Yr., Mo., Day) 1 MAR 1984									
15. PAGE COUNT 10													
16. SUPPLEMENTARY NOTATION													
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)										
FIELD	GROUP	SUB. GR.	1 MAR 1984										
19. ABSTRACT (Continue on reverse if necessary and identify by block number) This grant concerned the case of vector processors such as the CRAY-1 in the solution of sparse systems of equations. The study produced three major classifications of results. (1) Algorithms and related mathematical software for sparse solution on single processors (uniprocessors). (2) Preliminary projection of vector multiprocessor performance on linear algebra codes. (3) Cooperative work on vector sparse matrix algorithms with AFFDL for CFD and structures codes, and with UC/Berkeley on for circuit simulation.													
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS <input type="checkbox"/>			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED										
22a. NAME OF RESPONSIBLE INDIVIDUAL CPT John P. Thomas, Jr.			22b. TELEPHONE NUMBER (Include Area Code) (202) 767- 5026		22c. OFFICE SYMBOL NM								

I. INTRODUCTION

This grant concerned the case of vector processors such as the CRAY-1 in the solution of sparse systems of equations. The study produced three major classifications of results:

- (1) Algorithms and related mathematical software for sparse solution on single processors (uniprocessors).
- (2) Preliminary projection of vector multiprocessor performance on linear algebra codes.
- (3) Cooperative work on vector sparse matrix algorithms with AFFDL for CFD and structures codes, and with UC/Berkeley on for circuit simulation.



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II. TECHNICAL SUMMARY

A. Uniprocessor Studies

Figure 1 indicates how the single topic of general sparse matrix solution using scalar processors may be broken into specialized areas of study when implementation on vector architectures is considered.

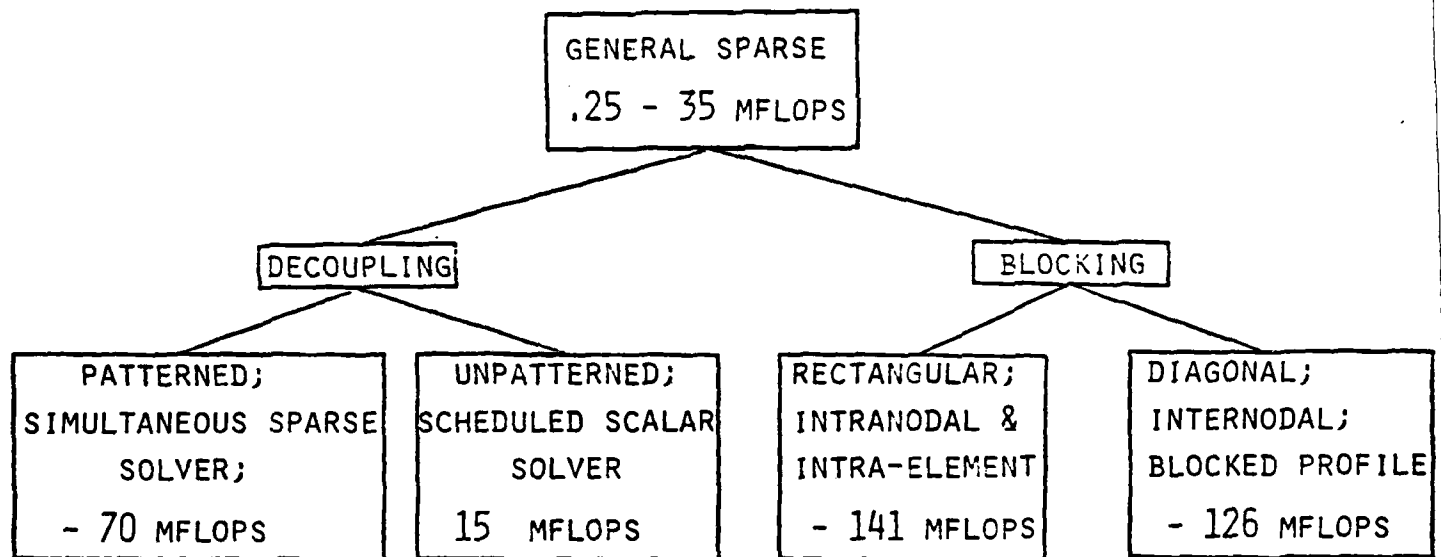
First, highly sparse matrices, usually representing ODE/algebraic-modeled systems, are easily decoupled by re-ordering. At a minimum, locally-decoupled equations may be solved in pipelined scalar mode; if the decoupled subsystems can be arranged (a) to have identical sparsity, and (b) to be stored a constant stride apart, then a simultaneous sparse solver may be invoked and a vector solution obtained.

As sparse systems become locally coupled - as occurs in finite element and finite difference problems - then vectors are easily defined within the coupled subsystems. It is worth making a further distinction between

- (a) intra-nodal or intra-element coupling, where the dimension of dense submatrices (and hence the vector length) is proportional to the number of unknowns/node or unknowns/finite element, and
- (b) inter-nodal or inter-element, where the coupling between grid nodes or finite elements determines the vector length.

Banded and profile matrices result from the latter. The associated vector lengths are the products of the number of unknowns/node (element) and the number of coupled nodes. These

Figure 1
HIERARCHY AND CLASSIFICATION OF SPARSE MATRIX SOFTWARE



lengths are therefore always longer than in the former case, so that common bandsolvers potentially offer the highest execution rate (MFLOPS) of any sparse solver applicable to finite element problems.

Algorithms and CRAY-1 mathematical software have been developed on this grant for

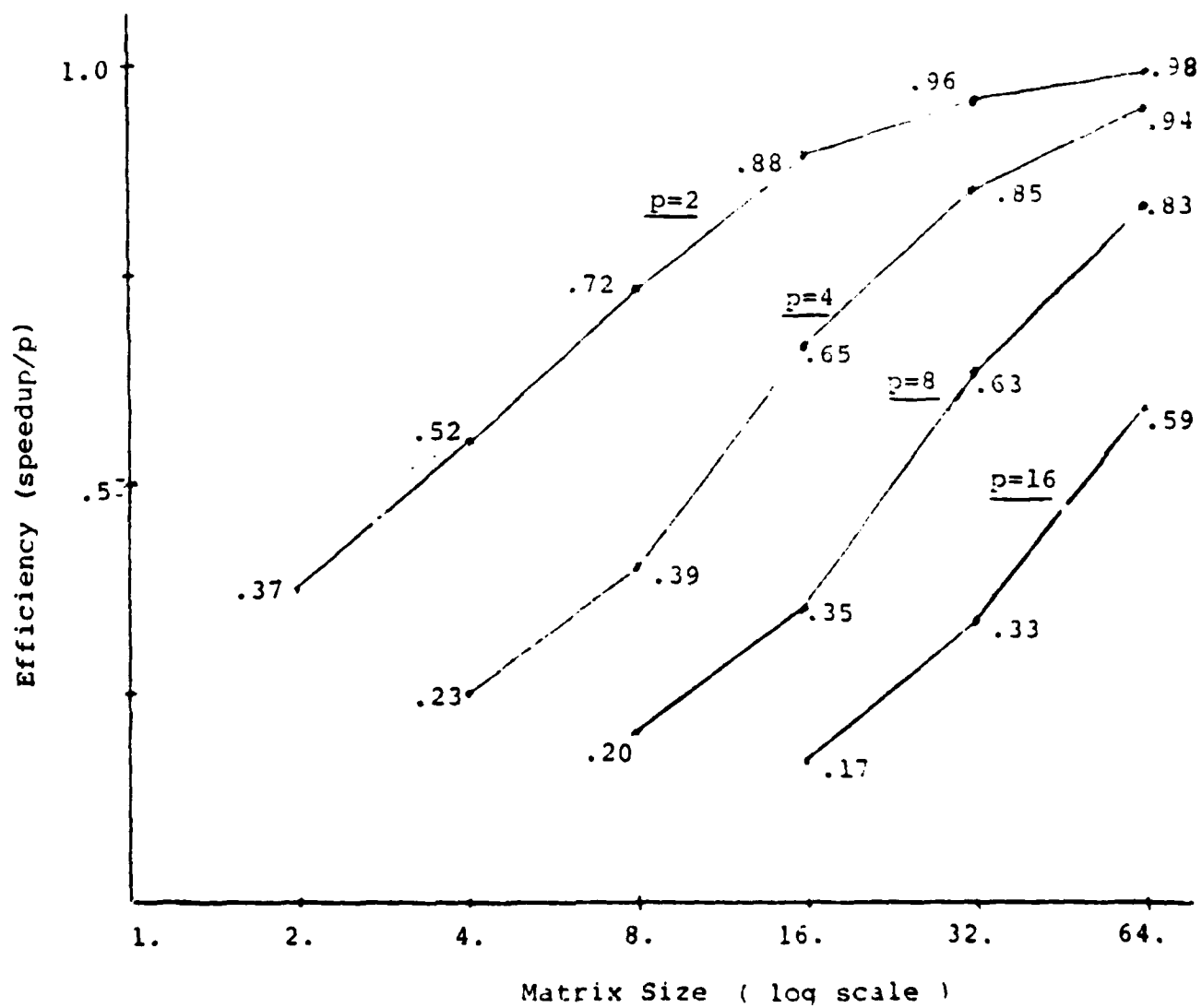
- (a) general sparse matrices, solved in traditional [15] and reorganized pipelined-scalar mode [7][16];
- (b) patterned sparse matrices, in conjunction with a vectorized electronic circuit analysis program [2];
- (c) blocked matrices arising from inter-nodal coupling [12];
- (d) both symmetric and unsymmetric banded and blocked-profile matrices [13][14];
- (e) simultaneous blocked tridiagonal systems, as arising in CFD [17].

Mathematical software resulting from these studies have been collected in a library [17].

B. Multiprocessor Studies

In the 1980's, to achieve GIGAFLOP performance with clock times in the range of 10 μ sec will require multiple vector processors executing cooperative tasks. The ability of extensions of the present CRAY family of processors to execute small tasks in an efficient manner was initially studied by developing a multi-processor extension of the CRAY-1 simulator [18]. Several algorithms were studied for the triangular factorization of small matrices. Figure 2 shows representative

Figure 2. Speedup of Microtasked Solution



efficiencies* for the cooperative LU factorization of matrices, with the number of processors (p) ranging from 2 to 16. These results were compared with 2-processor CRAY XMP timings, using an experimental in-house multitasking operating system at Cray Research, Inc.

This work will continue under a new AFOSR grant.

C. Air Force-related Applications

1. Electronics. A Ph.D. student from UC/Berkeley engaged in electronic circuit simulation was a visitor to our research group during the summer of 1980 to study the CRAY-1 and our sparse matrix research. Under subsequent Bell Laboratories and AFOSR auspices, he produced a series of papers and a vectorized version of the SPICE electronic circuits analysis program which achieves a 5-10:1 speedup on the CRAY-1. The speedup achieved from simultaneous sparse solution of subcircuit matrices- originally proposed in [2]- is the critical feature of this program.

2. Aerodynamics. Work was completed in 1980 on FDL-sponsored research on vectorization of computational fluid dynamics (CFD) codes. A four-day seminar on the general topic of vector processing was presented at FDL in 1980.

Air Force sponsored research on vectorized CFD algorithms has lead to related work currently sponsored by NASA/Ames Research Center.

3. Structures. From 1981 - 1983, two related finite

Efficiency (n)=(uniprocessor time)/(p(multiprocessor time))

element analysis and optimization codes from FDL were vectorized, under joint AFOSR-FDL sponsorship. By far the greatest speedup ($> 2000:1$) was due to vectorized banded equation solvers developed under a AFOSR sponsorship [14]. A report, including comparisons with NASTRAN, has been written on these results [20].

III. OTHER COUPLING AND PROFESSIONAL ACTIVITIES

A. Seminars

1. Washington State University (11/17/80)
2. University of California, Berkeley (11/19/80)
3. University of Texas, Austin (10/30/80)
4. 4-day seminar at AFFDL, (6/80)
5. Seminar at LANL (8/80)
6. Review of vector processing research, AFFDL (5/26/81; 7/14/83)
7. Review of the state-of-the-art in scientific computation at AFOSR (5/6/82).

B. Visiting Scientist and Consulting

1. Visiting scientist, AFFDL, to give instruction on algorithms for vector processing, and to study I/O problems associated with Navier-Stokes codes on the CYBER 203/205 (5/1/80 - 9/30/80).
2. Visiting scientist, LANL, on vectorized Monte Carlo (5/1/80 - 9/30/81).
3. Visiting scientist, LANL, on performance of PIC codes on CRAY-2 (5/1/83 - 8/1/83).
4. Visiting scientist, LANL, on task granularity on vector multiprocessors (10/1/83 - 4/30/84).

5. Industrial consultant, Mobil Research and Development, on the vectorization of 3-D diffusion codes associated with oil reservoir drilling and management (5/1/80 - 1/15/82).
6. Industrial consultant, Chevron Oil Field Research Co., on organization of vectorized sparse matrix algorithms (2/82) and on vector multiprocessors (12/83).
7. Consultant, LLNL, o vectorized Monte Carlo (5/1/82 - 9/30/83).
8. Visiting scientist, AFFDL, on vectorized structural analysis and optimization techniques (5/1/82 - 9/30/83).
9. Consultant, Cray Research, Inc. to profile projected CRAY-2 performance using instruction-level simulation (1/83 -).

C. Related Research

1. Principal investigator, NASA/ARC, on vectorization of computational chemistry codes (8/1/82 -).
2. Principal investigator, NASA/ARC, on preparation of scientific library for the CRAY-2 (12/5/83 -).

D. Professional

1. A one-week short course on Vector Processing was organized and presented at the University of Michigan during the summers of 1980, 1981, and 1982.
2. As an appointed member of a NASA Technical Review Board, an evaluation was made of proposals from Control Data Corporation and the Burroughs

Corporation for the \$100 million Numerical Aerodynamic Simulator (5/1/83 - 7/31/83).

3. Editor, IEEE Transaction on Computers, 8/1/82 - 12/31/83.

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Journal Articles

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Conference Publications

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